

$$556. \lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{\frac{1}{x}} \quad (a > 0, b > 0, c > 0).$$

$$557. \lim_{x \rightarrow 0} \left(\frac{a^{x+1} + b^{x+1} + c^{x+1}}{a + b + c} \right)^{\frac{1}{x}} \quad (a > 0, b > 0, c > 0).$$

$$558. \lim_{x \rightarrow 0} \left(\frac{a^{x^2} + b^{x^2}}{a^x + b^x} \right)^{\frac{1}{x}} \quad (a > 0, b > 0).$$

$$559. \lim_{x \rightarrow 0} \frac{a^{x^2} - b^{x^2}}{(a^x - b^x)^2} \quad (a > 0, b > 0).$$

$$560. \lim_{x \rightarrow a} \frac{a^{a^x} - a^{x^a}}{a^x - x^a} \quad (a > 0).$$

$$561. \text{ a) } \lim_{x \rightarrow -\infty} \frac{\ln(1 + 3^x)}{\ln(1 + 2^x)}; \quad \text{ b) } \lim_{x \rightarrow +\infty} \frac{\ln(1 + 3^x)}{\ln(1 + 2^x)}.$$

$$562. \lim_{x \rightarrow +\infty} \ln(1 + 2^x) \ln\left(1 + \frac{3}{x}\right).$$

$$563. \lim_{x \rightarrow 1} (1 - x) \log_x 2.$$

564. Dokázat, že

$$\lim_{x \rightarrow +\infty} \frac{x^n}{a^x} = 0 \quad (a > 1, n > 0).$$

565. Dokázat, že

$$\lim_{x \rightarrow +\infty} \frac{\log_a x}{x^\varepsilon} = 0 \quad (a > 1, \varepsilon > 0).$$

Najít limity

$$566. \text{ a) } \lim_{x \rightarrow 0} \frac{\ln(x^2 + e^x)}{\ln(x^4 + e^{2x})}; \quad \text{ b) } \lim_{x \rightarrow +\infty} \frac{\ln(x^2 + e^x)}{\ln(x^4 + e^{2x})}.$$

$$567. \lim_{x \rightarrow 0} \frac{\ln(1 + xe^x)}{\ln(x + \sqrt{1 + x^2})}.$$

$$568. \lim_{x \rightarrow +\infty} [(x + 2) \ln(x + 2) - 2(x + 1) \ln(x + 1) + x \ln x].$$

$$569. \lim_{x \rightarrow +0} \left[\ln(x \ln a) \cdot \ln\left(\frac{\ln ax}{\ln \frac{x}{a}}\right) \right] \quad (a > 1).$$

$$570. \lim_{x \rightarrow +\infty} \left(\ln \frac{x + \sqrt{x^2 + 1}}{x + \sqrt{x^2 - 1}} \cdot \ln^{-2} \frac{x + 1}{x - 1} \right).$$