ŠMF: STERN-GERLACHŮV EXPERIMENT

- 1922
- nehomog. mag. pole

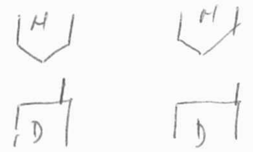
$\vec{\mu} = \vec{s}\,\frac{e\hbar}{m}$ mag. moment

$U = -\vec{\mu}\cdot\vec{B}$

$F = -\frac{\partial U}{\partial z} = -\mu_z \frac{\partial B}{\partial z}$

$s_z = \pm\frac{1}{2}$

- zarážka

1) 

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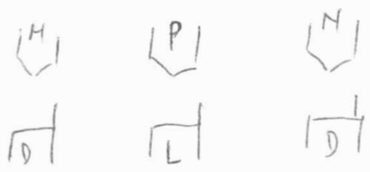
2) 

50% 50%

informace P/L likviduje informaci o H/D

- popis stavu: 1)

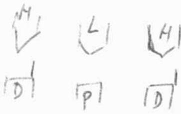
$|z\uparrow\rangle$ pravděpodobnost: $P_{z\uparrow z\uparrow} = |\langle z\uparrow|z\uparrow\rangle|^2 = 1$

$P_{z\uparrow z\downarrow} = |\langle z\uparrow|z\downarrow\rangle|^2 = 0$

$\Rightarrow$ např: $|z\uparrow\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $\langle z\uparrow| = (1,0)$

$|z\downarrow\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

2) $P_{z\uparrow z\downarrow} = |\langle z\uparrow|x\downarrow\rangle\langle x\downarrow|z\downarrow\rangle + \langle z\uparrow|x\uparrow\rangle\langle x\uparrow|z\downarrow\rangle|^2 = ?$



$|x\uparrow\rangle = \begin{pmatrix}a_1\\a_2\end{pmatrix}$ $|x\downarrow\rangle = \begin{pmatrix}b_1\\b_2\end{pmatrix}$

$|\langle z\uparrow|x\downarrow\rangle|^2 = \frac{1}{2}$ $|\langle z\downarrow, x\downarrow\rangle|^2 = \frac{1}{2}$ $\Rightarrow$ $|b_1|^2 = \frac{1}{2}$ $|b_2|^2 = \frac{1}{2}$

$|\langle z\uparrow|x\uparrow\rangle|^2 = \frac{1}{2}$ $|\langle z\downarrow, x\uparrow\rangle|^2 = \frac{1}{2}$ $|a_1|^2 = \frac{1}{2}$ $|a_2|^2 = \frac{1}{2}$

$|\langle x\uparrow|x\downarrow\rangle|^2 = 0$ $a_1 b_1 + a_2 b_2 = 0$

$|x\uparrow\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ $|x\downarrow\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

obdobně pro y:

$|y\uparrow\rangle = \begin{pmatrix}c_1\\c_2\end{pmatrix}$ $|\langle z\uparrow|y\uparrow\rangle|^2 = \frac{1}{2} \Rightarrow |c_1|^2 = \frac{1}{2}$ $|\langle y\uparrow|y\uparrow\rangle|^2 = 1 \Rightarrow |c_1|^2 + |c_2|^2 = 1$

$|y\downarrow\rangle = \begin{pmatrix}d_1\\d_2\end{pmatrix}$ $|\langle x\uparrow|y\uparrow\rangle|^2 = \frac{1}{2} \longrightarrow |c_1 + c_2|^2 = 1$

$|\langle x\uparrow|y\downarrow\rangle|^2 = \frac{1}{2}$ $|c_1 - c_2|^2 = 1$ $|y\uparrow\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$

$|\langle x\downarrow|y\uparrow\rangle|^2 = \frac{1}{2}$ $|y\downarrow\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$

$|\langle x\downarrow|y\downarrow\rangle|^2 = \frac{1}{2}$

lze pronásobit $e^{i\varphi}$ → volba fáze

$$\left|\langle z\uparrow|x\downarrow\rangle\langle x\downarrow|z\downarrow\rangle + \langle z\uparrow|x\uparrow\rangle\langle x\uparrow|z\downarrow\rangle\right|^2 = \left|\frac{1}{\sqrt{2}}(1,0)\begin{pmatrix}1\\-1\end{pmatrix}\cdot\frac{1}{\sqrt{2}}(1\ -1)\begin{pmatrix}0\\1\end{pmatrix} + (0,1)\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\cdot\right.$$

$$\left.\cdot\frac{1}{\sqrt{2}}(1,1)\begin{pmatrix}0\\1\end{pmatrix}\right|^2 = \left|-\frac{1}{2}+\frac{1}{2}\right|^2 = 0$$

$$\left|\langle z\uparrow|x\uparrow\rangle\langle x\uparrow|z\downarrow\rangle\right|^2 = \left|(0,1)\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\cdot\frac{1}{\sqrt{2}}(1,1)\begin{pmatrix}0\\1\end{pmatrix}\right|^2 = \frac{1}{2^2} = \frac{1}{4}$$

Dú totéž s y.

$$|x\uparrow\rangle\langle x\uparrow| + |x\downarrow\rangle\langle x\downarrow| = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\frac{1}{\sqrt{2}}(1\ 1) + \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\frac{1}{\sqrt{2}}(1\ -1) = \frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1&-1\\-1&1\end{pmatrix} =$$

$$= \begin{pmatrix}1&0\\0&1\end{pmatrix}$$

Dú ostatní!

$$\hat{A}_z = \frac{1}{2}|z\uparrow\rangle\langle z\uparrow| - \frac{1}{2}|z\downarrow\rangle\langle z\downarrow| = \underbrace{\frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}}_{\text{Pauliho matice}}$$

Dú zbytek