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Review text:

The existence of a unitary quantum system defined by a complex, typically PT-symmetric, potential $V(x) \neq V^*(x)$ sounds like a paradox. The clarification of the paradox is easy: The apparent non-hermiticity of the Hamiltonian $H = p^2 + V(x)$ is revealed as connected with its definition in a “fixed” or “first” Hilbert space $\mathcal{H}^{(F)}$ (chosen, most usually, as the most common and “friendly” but, in this context, “false” $\mathbf{L}^2(\mathbf{R})$). Naturally, the puzzle disappears and the hidden (better called crypto) hermiticity emerges in another, “second” or “standard” Hilbert space $\mathcal{H}^{(S)}$.

For similar quantum models the imaginary part of the corresponding CLASSICAL Hamiltonian can be treated as the first class constraint (FCC, cf. [5] where, e.g., the existence of a very interesting and slightly unconventional resulting symplectic structure of the corresponding classical dynamical system has been revealed). For an explicit and extensive illustration of some other physical as well as mathematical details, the one-dimensional harmonic-oscillator solvable example has been chosen and discussed by Smilga [4]. I.a., he emphasized that several different nontrivial quantum models can in turn be formulated starting from a given classical problem with FCCs.

The present MS complements the latter study by its extension to two and three dimensions, emphasizing that the possible presence of further observables (here, angular momenta) leads to the natural emergence of further FCCs. Unfortunately, the authors decided to present just a small and rather formalistic sample of consequences of this interesting fact, with their rest to be “reported elsewhere”.