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Review text:

For a typical mathematician the title of this paper must sound incomprehensible because he/she knows that in a consistent quantum theory the Stone's theorem must be obeyed. In other words the Hamiltonian H of a (stable) quantum system must *always* be (essentially) self-adjoint. An easy key to the resolution of this apparent paradox lies in the (implicit but, recently, increasingly popular) *parallel* representation of a given quantum system in *several* (in general, unitarily non-equivalent) Hilbert spaces $\mathcal{H}^{(1)}, \mathcal{H}^{(2)}, \mathcal{H}^{(3)}, \ldots$.

Typically, the operator H acquires a particularly simple (i.e., say, easily diagonalizable) form in $\mathcal{H}^{(1)}$ where it remains, by assumption, manifestly non-self-adjoint. The trick (summarized and (re)explained, more thoroughly, elsewhere - cf., e.g., M. Znojil, SIGMA 5 (2009) 001 [19 pages]) is based on a non-unitary generalization of the Fourier-type mappings. This mapping (denoted as $exp(-\mathcal{Q}/2)$ in MS under consideration, or by the symbol Ω in *loc. cit.*) "pulls back" our H into a family of amended, "physical" (although not always mutually unitarily equivalent) Hilbert spaces $\mathcal{H}^{(2)}, \mathcal{H}^{(3)}, \ldots$. In these new spaces the Hamiltonian becomes safely self-adjoint and Hermitian. For this reason, *all* our Hamiltonians *are* Hermitian in a suitable space (so, one shouldn't call them non-Hermitian but rather "cryptohermitian" at most).

In a typical "computationally friendly" model of such a class we start working in the "false" space $\mathcal{H}^{(1)}$ chosen as the Hilbert space of quadratically integrable functions of N real variables, $\mathcal{H}^{(1)} \equiv L^2(I\!R^N)$. In parallel we restrict our attention to Hamiltonians $H = T + V$ where the first, linear-differential-operator component $T = \sum a_n \partial_n^2$ represents kinetic energy. In this context the necessity of moving to "physical" spaces $\mathcal{H}^{(2)}, \mathcal{H}^{(3)}, \ldots$ is, typically, evoked by the

transition to a broader class of interactions (given, say, by a *complex* V in $\mathcal{H}^{(1)}$).

In this context the present MS pays attention to several solvable examples where the energies remain real for a weak coupling between some well separated subspaces of $\mathcal{H}^{(1)}$ where V is chosen as real and complexified, respectively.