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**Reviewer:** Znojil, Miloslav

**Reviewer number:**

**Address:**
NPI ASCR,
250 68 Rez,
Czech Republic
znojil@ujf.cas.cz

**Author:** Mostafazadeh, Ali; Mehri-Dehnavi, Hossein

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**Review text:**

One of the authors of this - extremely interesting - paper [P1] produces his results so quickly that the citation of the completed subsequent paper [P2] is already available as ref. [19] in [P1] (also note that another progress report [P3] is announced at the end of the text). This speed is, unfortunately, felt. I.a., it forces me to make this extended abstract a bit more explicatory (alternatively, interested readers might skip my forthcoming explanations and have a look at the more matured next-step progress report [P2] instead).

Firstly: paper [P1] is about the structure of the spectrum of the 1D Hamiltonian with the complex double-well delta-function potential given in Eq. (3). In this sense, even the title would deserve an amendment (its dominant first line carries just a complementary information). Personally I would also like to attract attention to my own paper on the same model (M. Znojil, J. Phys. A: Math. Gen. 36 (2003) 7639-48) and to its early practical use in connection with certain strong-coupling expansions. Nowadays, the area of applications of similar models is much broader of course, involving even experiments in nonlinear optics (so that references [3,4] would deserve an update).

Secondly, also the abstract admits an ambiguous reading. Certainly, the paper's main message is not about a 1D quantum problem of scattering but rather about its non-unitary (though still very interesting) mathematical generalization. This class of methodically relevant models (which cannot directly be assigned the standard quantum-mechanical meaning in general) has been called "effective", e.g., by the author of the second item in ref. [13] (say, in his other, much more relevant paper with coordinate D 77 065023 merely replaced by D

78 065032).

All this being said let me emphasize that although the whole text (having, by the way, its half-line-model immediate predecessor in Ref. [20]) makes an impression of a hastily finished draft and although it is difficult to read it in places, it certainly deserves to be read. The main reason is that it offers an almost exhaustive account of the key features of the spectrum of the four-parametric Hamiltonian of Eq. (3). They are illustrated by 11 figures showing, predominantly, the position of spectral singularities (= real zeros of Jost functions) and of "bound states" (= certain complex zeros of Jost functions), the knowledge of which can lead immediately to the specification of the boundaries of (quasi-)Hermiticity of the model.