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Short title: Small-ϵ behavior of the non-Hermitian PT-symmetric Hamiltonian $H = p^2 + x^2(ix)^\epsilon$.

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Review text:

A long-missing contribution to the branch of constructive quantum mechanics a.k.a. cryptohermitian, quasi-Hermitian, PT-symmetric or pseudo-Hermitian quantum mechanics. In this approach the building of new tractable phenomenological models is based on their simplification via a nonstandard (i.e., nontrivial, metric-mediated and, in general, strongly ambiguous re-) definition of the dual elements in the Hilbert space of states [i.e., in Dirac's terminology, of the so called bra vectors - a more detailed version of this abstract explanation can be found in [11] or in my text in SIGMA 5 (2009), 001]. Concerning the particular, illustrative anharmonic-oscillator example given in the title, some of its puzzling features are already known for more than thirty years. The history [partly told to me by Gabriel Alvarez, cf. also his comprehensive text in J. Phys. A: Math. Gen. 27 (1995) 4589] started, perhaps, by the half-forgotten perturbation-theory paper on the asymptotically purely imaginary anharmonicity where, asymptotically, $\epsilon = 1$, cf. Caliceti et al, Comm. Math. Phys. 75 (1980) 51. Still, the paper under consideration seems to be the first successful attempt introducing the above-mentioned definition via perturbation series using the smallness of exponent ϵ. Psychologically, this is a slightly surprising fact since it was precisely Bender et al who developed the appropriate method (called $\delta-$expansions, cf. ref. [13]) five years earlier than their attention has been re-directed to the apparently non-Hermitian Hamiltonians in question (by Daniel Bessis, cf. footnote [12]).