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**Review text:**

After cca 30 years the group of mathematical physicists from Bologna and Modena returns to their beloved problem of the Padé (Stieltjes) summability of the (cubic) anharmonic oscillator energy levels and proves it. They add a simple proof of the absence of non-perturbative eigenvalues and of the related Bessis - Zinn-Justin conjecture (concerning the reality and also the simplicity and positivity of energy levels). Relations are finally found between zeros of wave functions and the semiclassical quantization rules.

It is worth adding that during the last ten years the model in question acquired a lot of popularity in quantum physics (cf. also the fairly incomplete list of references), especially because its apparently non-Hermitian Hamiltonian $H_\alpha(\beta)$ (cf. eq. (1)) proves strictly self-adjoint in an *ad hoc* Hilbert space $\mathcal{H}^{(S)}$ where, in the notation of our recent more detailed review paper (M. Znojil, Three-Hilbert-space formulation of Quantum Mechanics, SIGMA 5 (2009), 001, doi: 10.3842/SIGMA.2009.001), the superscript $^{(S)}$ may be read as standing for either "standardized" or "sophisticated". This means that in $\mathcal{H}^{(S)}$ the "correct physical" Hermitian conjugation operation $\mathcal{T}^{(S)} : |\psi\rangle \to \langle\!\langle\psi|$ must be introduced as defined in terms of a nontrivial (often called "non-Dirac") metric operator $\Theta \neq I$. This yields $\langle\!\langle\psi| := \langle\psi|\Theta$ . As a consequence the apparently non-Hermitian Hamiltonian $H_\alpha(\beta) \neq [H_\alpha(\beta)]^\dagger$ may be perceived as an isospectral transform [sometimes called a Dyson's non-unitary map – cf. F. G. Scholtz, H. B. Geyer and F. J. W. Hahne, Quasi-Hermitian Operators in Quantum Mechanics and the Variational Principle, Ann. Phys. (NY) 213 (1992) 74-101] of a "true" or "physical" self-adjoint Hamiltonian operator $h = h^\dagger$. The readers are recommended to find [e.g., in A. Mostafazadeh, PT-Symmetric Cubic Anharmonic Oscillator as a Physical Model, J. Phys. A: Math. Gen. 38 (2005)

65576570], the explicit perturbative approximate representation of the latter, drastically more complicated “standard” self-adjoint Hamiltonian in question.