This is a review text file submitted electronically to MR.

Reviewer: Znojil, Miloslav

Reviewer number:

Address:
NPI ASCR,
250 68 Rez,
Czech Republic
znojil@ujf.cas.cz

Author: Ghosh, Pijush K.

Short title: On the construction of a pseudo-Hermitian quantum system with a pre-determined metric in the Hilbert space.

MR Number: 2599802

Primary classification: 81Q12

Secondary classification(s): 81Q10 81Q05 46C20 47B50

Review text:

In the recent literature on applications of the standard formalism of quantum mechanics of unitarily evolving systems one often finds papers speaking about "non-Hermitian" Hamiltonians H with real spectra which generate the evolution. In a way which I thoroughly explained elsewhere (cf. M. Znojil, "Three-Hilbert-space formulation of Quantum Mechanics", SIGMA 5 (2009), 001, doi: 10.3842/SIGMA.2009.001, arXiv: 0901.0700), this is a pure misunderstanding. In ALL of these cases, the operator H merely appears non-self-adjoint in some ill-chosen, auxiliary, "false" representation of the Hilbert space of states (denoted as $\mathcal{H}_D$ by Ghosh). In parallel, our H remains safely self-adjoint in a certain "standard" textbook representation of the physical Hilbert space of states (denoted as $\mathcal{H}_{\eta_+}$ by Ghosh).

Let us emphasize that the main and only source of misunderstandings of this type is that for certain strange reasons people really love working in the "friendly", auxiliary representation space $\mathcal{H}_D$ which is manifestly unphysical.

Via his paper, Ghosh tries to make these people happy by listing several (roughly counting, six different) illustrative examples of H for which the correct representation space $\mathcal{H}_{\eta_+}$ still seems to remain known and sufficiently simple. In the language of physics, these examples are based on the use of specific Bogoliubov transformations and include single particle models (harmonic oscillator and Stark and Zeeman effects) as well as many-body systems including also various spin chains and/or, last but not least, LipkinMeshkovGlick model. Involuntarily, the author performs, in this manner, a full-circle return to the many-body example given also as illustrative in the 1992 paper [4] by "fathers founders" of the whole business.

The paper is well written and instructive. Its author selects and defines his space $\mathcal{H}_{\eta_+}$ in advance (in fact, via eqs. (2) and (6)). In the language of mathematics this would immediately lead to some doubts and questions about the (mathematically necessary though often ignored conditions of) boundedness of the metric η_+ of eq. (6) and of its inverse. A further recommended reading and a more extensive discussion related to the latter and similar subtleties may be recommended, say, in its first formulation "Quasi-Hermiticity in infinite-dimensional Hilbert spaces" by R. Kretschmer and L. Szymanowski in Phys. Lett. A 325 (2004) 112 - 117.