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Short title: Bloch decomposition-based Gaussian beam method for the Schrodinger equation with periodic potentials.

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Review text:

In the context of solid-state physics the linear Schroedinger equation is considered describing, typically, the motion of electrons in potentials generated by ionic cores. The authors consider the one-dimensional model with the two-scale WKB initial condition and with potentials periodic on the lattice $\Gamma = 2\pi Z$.

In the dynamical regime where the direct numerical simulations are prohibitively expensive and where the presence of many caustics reduces the accuracy of semiclassical methods (i.e., e.g., of the time-splitting spectral method of Ref. [3]) the authors generalize the Gaussian beam method introduced in 2008 [23]. The idea is to allow the phase function to be complex, still admitting that the solution has a Gaussian profile for each of a few (non-crossing) Bloch bands.

The necessary mesh size proves much coarser, allowed to be of the square-root order in the small semiclassical parameter. This favorably compares with the mere first-order mesh characterizing the recent Bloch-decomposition-based techniques of Refs. [18, 19, 20].

Several (e.g., insulator) numerical examples illustrate the accuracy and confirm the efficiency of the new method, with promising future applications, especially in the higher spatial dimensions.