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Review text:

From the point of view of relativistic quantum mechanics, the key target of this study is the so called Klein paradox: Everybody knows – or may read in refs. [50] - [53] – that for the apparently confining potential $V(x) = \omega^2 x^2$ (and the like – in units $m/2 = 1$) the Dirac or Klein-Gordon spectrum of "energies" appears, paradoxically, continuous and unbounded. Still, in a way explained by the first coauthor in ref. [27] one can construct the so called metastable states and arrive at many useful experimental predictions (typically, of the pair-production rates).

From the point of view of pure mathematics, the main methodical tool offered by the paper under review (which may be also read as a certain continuation of their previous paper [28]) lies in the change of perspective mediated, in essence, by certain suitable isospectral as well as non-isospectral changes of the Hamiltonian. This idea dates back to the 18 years old ref. [37] coauthored by the second present coauthor and, by the way, not too often known or cited in a recent series of rediscoveries of the concept of PT-symmetry. In this context let me also add, marginally (cf. ref. [1]), that it was a true pleasure of mine to see the new example of efficiency of the concept of the – here, "Titchmarsh-operator" or "associated" – energy-dependent and non-Hermitian though mutually conjugate PT-symmetric Hamiltonians $H_{\pm}$.

The resulting message is extremely interesting: the use of auxiliary (in some sense, just partially or asymptotically isospectral) $H_{\pm}$ (as well as the use of sophisticated methods of its numerical analysis) is shown to provide a truly reliable approximate localization of the relativistic resonances as well as the strict correspondence of the metastable states to the bound states in non-relativistic limit.

An important encouragement is provided for all of the recent efforts aimed at extraction of information from the analyticity (or, more explicitly, from a complex-dilatability) of certain "useful" operators with continuous spectra. In this sense the paper is also remarkable and non-main-stream by its *admitting* the natural *dissipative* nature of quantum evolution. Last but not least, the mathematically rigorous and exhaustive though still legible style of the text might offer a very promising bridge between the physics- and mathematics-oriented communities.