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Review text:

The use of point interactions makes the Schrödinger equation of quantum mechanics (say, on half-axis) exactly solvable (readers are recommended to consult the comprehensive 2005 monograph [1] by Albeverio et al). Once one assumes their special case of derivatives of Dirac delta functions centered at a strictly increasing unbounded sequence $X = \{x_k\}_{k=1}^{\infty}$ and endowed with respective couplings β_k, one usually consults the Gesztesy's and Holden's 1987 paper [18] for basic information. In this context, the present authors propose to employ, in a way paralleling ref. [5], the language of forms. Having resolved the key technical puzzle of making the related forms closable (by splitting them in two at each center k) they are able to spot the key differences between δ and δ' point interactions emphasizing that in the latter case [a] the role of a free "unperturbed" Hamiltonian is taken by a direct sum of its Neumann's analogues in $L^2(x_{k-1}, x_k)$ (cf. also [16]), [b] one can parallel several $\delta-$interaction results known from the Glazman's and Wiedenholtz' monographs [19,40] and Molchanov's paper [36] deducing, in particular, the self-adjointness from the semiboundedness. The criteria of semiboundedness are then studied and used to find several (though still not all) necessary and sufficient conditions of the spectral discreteness and essentialness. As a premium, multiple interesting cases of operators with exotic essential spectra are finally discussed and sampled.