This is a review submitted to Mathematical Reviews/MathSciNet.

Reviewer Name: Znojil, M.

Mathematical Reviews/MathSciNet Reviewer Number: 13388

Address:
Theory Group
NPI ASCR
250 68 Řež u Prahy
CZECH REPUBLIC
znojil@ujf.cas.cz

Author: Bender, Carl M.; Fring, Andreas; Komijani, Javad

Title: Nonlinear eigenvalue problems.

MR Number: MR3216779

Primary classification: 34A12

Secondary classification(s): 34C15 34D05 34E20 34M55

Review text:

The title is slightly misleading: in the first three sections of this four-section text the authors study just the ordinary differential Cauchy problem $y'(x) = cos[\pi xy(x)]$, with emphasis upon its numerical and asymptotic analysis. The authors feel guided by a resemblance of solutions $y(x)$ (and, in particular, of their separatrices) to harmonic-oscillator bound-state wave functions and, in particular, to their Sturm-Liouville oscillation-theorem-based qualitative features. In this picture the role of the growth of the harmonic-oscillator-like eigenenergies $E = E_n, n = 0, 1, \ldots$ is transferred to the growth of the initial-value quantities $y(0) = a_n, n = 0, 1, \ldots$. The nonlinear-problem parallel to the WKB $n \gg 1$ approximation is then developed as a hidden linearization yielding the leading-order formula $a_n \sim 2^{1/3}\sqrt{2n} \approx 1.781797436\sqrt{n}$ (accompanied, incidentally, by the conjecture of coincidence of the number $1.781797436\ldots$ with a Hayman's universal power-series constant of ref. [7]). Finally, the acquired methodical experience is fructified in a proposal of the author's future project covering the first Painlevé transcendent equation. A few supportive numerical experiments are performed yielding, *mutatis mutandis*, the Painlevé-related leading-order formula $a_n \sim Cn^{3/5}$ where $C \approx 4.284$.