This is a review submitted to Mathematical Reviews/MathSciNet.

Reviewer Name: Znojil, M.

Mathematical Reviews/MathSciNet Reviewer Number: 13388

Address:
Theory Group
NPI ASCR
250 68 Řež u Prahy
CZECH REPUBLIC
znojil@ujf.cas.cz

Author: Feranchuk, Ilya; Ivanov, Alexey; Le, Van-Hoang; Ulyanenkov, Alexander

Title: Non-perturbative description of quantum systems.

MR Number: MR3308409

Primary classification: 81-02

Secondary classification(s): 81Q05 35J10 35Q40 81Q15 81V45

Review text:

The choice of the title of the book is certainly unfortunate. Indeed, when I showed this title to a number of my colleagues, many of them immediately associated the book with the two years old Strocchi's excellent monograph on non-perturbative quantum field theory. No surprise that they felt frustrated by the misunderstanding. Moreover, those of them who still, with obvious curiosity opened the book, reported just an additional disappointment. What they did expect was something like functions e^{-1/x^2} but what they truly got was just the split of Hamiltonian H into the very traditional sum $H = H_0 + V$. From their only slightly remote perspective, nothing beyond perturbation theory.

The use of the misleading title acquires additional flavor of disbelief when one notices that the authors dedicated their very carefully prepared, well organized review to the memory of Lev Ivanovich Komarov who was one of the pioneers of the development of what the book is truly about, viz., of the so called (and cleverly called) "operator method" (OM) which is a fairly universal tool for the solution of an impressively broad class of various bound-state Schroedinger equations $H\,|\psi_n\rangle = E_n|\psi_n\rangle$.

Probably, the overambitious authors intended to promise more than they could deliver. It is a pity. Indeed, the book is comprehensive. It deserves full attention of the appropriate readership and, first of all, of all of the researchers who work in the field of perturbation theory and of its practical applications. Naturally, in order to attract their attention a better choice of the title would also deserve to be paralleled by a more open-minded preface and Chapter 1. Frankly: in place of reading the highly philosophical introductory text about the "Capabilities of Approximate Methods in Quantum Theory" in Chapter 1 I would personally prefer to start my reading from a concise outline of the basic OM ideas.

Fortunately, in this context I feel strongly privileged by my age (in 1982 I read the pioneering Feranchuk-Komarov paper in Physics Letters immediately after it appeared), by my interests (I read this text - and many of its follow-ups - with genuine appreciation) and by my scientific contacts (let me add here my special thanks, for numerous debates and for his prevailingly enthusiastic praise of the merits of the subject, to Marcelo Fernández). So let me now try to save time of the readers *in spe* (i.e., of many theoretical physicists and quantum chemists) and to fill, partially, the gap. With apologies that my recommendation to read the book will be too concise (I shall skip a few rather essential subtleties like, e.g., a transition from wave functions to statistical operators, etc) and not always sufficiently rigorous (after all, we have the book!).

In OM, in essence (i.e., in the sufficiently simple applications at least), the Hamiltonian H is assumed given in a generic coordinate-, momentum- and coupling-dependent form $H(x, p, \lambda)$. Immediately, this (i.e., mostly, partial differential) operator is being transformed into an algebraic form defined in terms of the usual creation and annihilation operators $a^\dagger$ and a, respectively.

The latter step allows one to introduce a new, auxiliary, "mass-like" free parameter ω, to be assigned a variational-like optimization role later on. In the subsequent, key step of the construction one separates $H = H_0 + V$ where H_0 is "diagonal" (i.e., it depends solely on the number operator $n = a^\dagger a$) while the rest of H is a "non-perturbative perturbation" $V = H - H_0$.

The rest of the story is to be found in the book. There are lots of details, lots of clever tricks and lots of illustrative examples including many numerical tables and colored figures. Interested (plus, hopefully, hereby converted) readers may either swallow all the text or pick up some raisins. These are presented, in a comparative independence, in Chapters number two (about the method), three (mainly about various 1D anharmonicities), four (where the temperature enters the game), five (here, more degrees of freedom are taken into account), six (remarkable: magnetic field is now admitted) plus 7 and 8 (about atoms) and the last one (mainly about polaron, perceived as a representative of systems possessing infinitely many degrees of freedom).

Having read Chapter 9 the reader might miss a Summary or Outlook. But this is just proper time to return and indulge in the Introduction. One may now properly appreciate the statements about the prominent features of OM in which the mathematical concept of smallness of perturbations is certainly *different* from the traditional physicist's perception of what should be small in the Hamiltonian. With a final advice added at the end: the readers should take the dramatic description of the drawbacks of the existing alternative methods *cum grano salis*.