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Review text:

One of paradoxes of the rapidly developing non-Hermitian (and sometimes called PT symmetric) quantum mechanics (where non-self-adjoint Hamiltonians may still generate real spectra) is that its formulation has been motivated and guided by a fairly difficult particular example, viz., by the purely imaginary cubic oscillator [for example, I was personally attracted to this problem in private discussion with Daniel Bessis in the context of field theory in 1992. Let me also note that the proof of reality of its spectrum has only been given by Dorey et al, almost ten years later, in J. Phys. A: Math. Gen. 34 (2001) 5679-5704]. The next step of development was illustrated by the less difficult but still merely incompletely solvable and partially complex quartic oscillator example as formulated by V. Buslaev and V. Grecchi in J. Phys. A: Math. Gen. 26 (1993) 5541 and solved by C. M. Bender and S. Boettcher in J. Phys. A: Math. Gen. 31 (1998) L273 at L=0 [and by myself in J. Phys. A: Math. Gen. 33 (2000) 4203-4211 at all L]. Near the very end of the last century, the time proved ripe, at last, for my note [Phys. Lett. A 259 (1999) 220] about the expectable easiness of the PT symmetric complexification of the harmonic oscillator. In this context, I am warmly recommending the present Bagchi's and Quesne's text which should have come first. It marks the start of the new millenium and puts many a puzzle in a right mathematical perspective by taking "all" the so called exactly solvable PT symmetric Hamiltonians as Casimir operators in a suitable representation of the complex Lie algebra sl(2,C). One of the most appealing features of this formulation of the problem of bound states is the natural manner in which one can treat the existence of the so called unavoided crossings of energies (for me, one of the biggest surprizes encountered in preparation of my above-mentioned letter), i.e., the mergers of certain energy pairs which may open, in particular, also their subsequent transition to complex plane (organized always in the mutually conjugate pairs).

The main body of the text is a carefully presented handbook of formulae. My own above-mentioned harmonic-oscillator Laguerre-polynomial solutions emerge here in their Morse-potential re-arragement (equivalent after an easy change of variables). In the case of the remaining two exactly solvable cases (a.k.a. Scarf II and Poeschl-Teller oscillators with Jacobi-polynomial solutions), the authors claim that their solutions disagree with the older formulae of refs. [8] and [24] (by Levai and me). I must have a look at it right away.