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Reviewer: Znojil, Miloslav

Reviewer number: 13388

Address:
NPI AV CR,
250 68 Rez,
Czech Republic
znojil@ujf.cas.cz

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Review text:

Let us remind the reader about the two apparently independent elementary facts concerning the quantum mechanics (QM) in one dimension (of course, one can extend them with certain care to more dimensions for systems, say, with central symmetry). Firstly, *ANY physical* system of this type is specified by the ordinary linear differential Schroedinger equation of the second order. Secondly, *ANY mathematical* differential equation of this type possesses the so called "fundamental" set of the two linearly independent solutions. Hence, one would *a priori* expect that *ANY* (or at least a vast majority of the) popular Lie-algebraic interpretation(s) of the QM states (speaking about "symmetry" in physics and about "representations" in mathematics) will work with these *fundamental* solutions. Rather surprisingly, it is not so. Only the *pioneering* paper in question *starts* doing so.

In a consequent acceptance of such a basic idea, the authors review the relevant definitions and restrict their attention to the certain spectrum-generating algebras "for Sturmians" (for which, roughly speaking, the roles of the energy and coupling of a bound state are being exchanged), i.e., in their terminology, to the so called "potential algebras". They, in brief, succeeded in replacing the current so(2,1) (etc) by so(2,2) (etc) and bring a new perspective to the whole business.

Their main technical step lies in an "extension" of the concept of the ladder-type operators, i.e., alternatively, of the first-order differential factors of Infeld and Hull in ref. [1], or of the equivalent supersymmetric (SUSY) charges of review [2] etc. The authors employ, for this purpose, the complexified QM formalism of Bender and Boettcher (ref. [15] from 1998) although the construction itself seems to survive even within a much less “revolutionary” PT symmetric QM framework as proposed by Buslaev and Grecchi in J. Phys. A: Math. Gen., vol. 26, pp. 5541 - 5549 in 1993 (indeed, the latter recipe seems to suffice for the necessary regularization of the purely Hermitian SUSY-type constructions about which I just gave more details in the LANL preprint arXiv: hep-th/0209262).

In detail, a sufficiently flexible Ansatz for the generators is postulated in a differential form of the first order, constrained by the appropriate requirement of their compatibility with the commutation relations. This gives the class of the eligible algebras and Hamiltonians (= the first Casimirs: the second one always happens to vanish). The free-parameter functions are then fitted in systematic manner and all the formulae are worked out for the Scarf’s (regular) and Poeschel’s and Teller’s (singular) hyperbolic potentials.