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**Review text:**

The concept of “exact solvability” in quantum mechanics is fairly vague and, in extremis, it may cover just harmonic oscillator for which virtually every formula of practical relevance is available in many textbooks. In one dimension, harmonic oscillator finds its “almost equal” solvable partner in the so called Scarf II potential (= a superposition of the tangens hyperbolicus of the coordinate with the inverse cosine hyperbolicus squared). The basic philosophy of the paper in question is to add more support to the latter statement and hope. Its authors study the overlaps of the wave functions of bound states and derive their representation in closed form. Many of these results are presented here, rather amazingly, for the first time. The reason of this success may be seen in the accepted strategy which generalizes the problem first, by extending the permitted range of the coupling constants to the whole complex plane. The resulting formulae for the overlaps are then shown compatible not only with the traditional Hilbert-space special postulates of standard quantum mechanics (where the potential in question must be chosen real) but also with the modified Krein-space framework of the certain pseudo-self-adjoint extension of the standard quantum theory (the study of which is increasingly popular under the name of PT symmetric quantum mechanics these days). Several unusual aspects of the model in its latter non-Hermitian extension are emphasized incorporating, e.g., the bi-orthogonality and pseudo-normalizability of the states, the role of their quasi-parity and the structure of the spectrum which may be purely real or contain complex conjugate energy pairs in the regime with the so called broken PT symmetry.