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LOGARITHMIC WAVE EQUATION: ORIGINS AND APPLICATIONS

Quantum wave equation with logarithmic nonlinearity is introduced from the viewpoint of motivation and arguments arising from the conventional (linear) quantum formalism and theory of quantum liquids.

Keywords: quantum mechanics, wave equation, quantum nonlinear phenomena

1. Introduction

We consider the nonlinear logarithmic Schrödinger equation (LSE)

$$i\partial_t \psi(\vec{x},t) = (-\Delta + V_{LSE})\psi(\vec{x},t), \quad V_{LSE} = -b \ln|\psi(\vec{x},t)|^2, \frac{1}{2} \tag{1}$$

where Δ is the d-dimensional Laplacian, b is a real-valued constant; the equation has the wavefunction solutions $\psi \in L^2(\mathfrak{R}^d)$. This equation, as well as its relativistic analogue (which is obtained by replacing the derivative part with the d'Alembert operator), finds numerous applications in extensions of quantum mechanics [1, 2], physics of quantum fields and particles [3, 4, 5, 6, 7], optics and transport or diffusion phenomena [8, 9, 10], nuclear physics [11, 12], theory of dissipative systems and quantum information [13, 14, 15, 16, 17, 18], theory of superfluidity [19, 20, 21] and effective models of physical vacuum and classical and quantum gravity [22, 23, 24, 25].

The physical meaning of the solutions ψ and proper interpretation of their dynamics are determined by a phenomenological background of a given application. Still, the generic mathematical features of Eq. (1) (and, first of all, the logarithmic form of its nonlinearity) can be shared. For the purposes of illustration one can select, as a starting point, the context of (linear) quantum mechanics. In its de-Broglie's and Bohm's formulation [26] one often uses the Madelung representation of a wavefunction,

$$\psi(\vec{x},t) = R(\vec{x},t)\exp[iS(\vec{x},t)], \tag{2}$$

where both functions $R(\vec{x},t)$ and $S(\vec{x},t)$ are real-valued, see also ref. [28] for more commentaries. If one inserts this ansatz back into Eq. (1) then one reveals that the nonlinear interaction term

$$V_{LSE} \sim \ln|\psi(\vec{x},t)|^2 = 2\ln R(\vec{x},t) \tag{3}$$

becomes independent of the phase $S(\vec{x},t)$, which physical meaning and consequences are worth of studying. We shall begin from the standard linear quantum theory in which we found several arguments for introducing LSE, see section 2. The motivations coming from theory of quantum liquids are discussed in section 3. Summary and concluding discussions are given in section 4.

2. Origins in the linear theory

In this section, we show how LSE's usage can be motivated by a conventional (linear) quantum mechanics.

In the Schrödinger picture [27], the evolution of quantum systems is commonly described by the Schrödinger equation

$$i\partial_t \psi(\vec{x},t) = \hat{H}\psi(\vec{x},t), \quad \hat{H} = -\Delta + V(\vec{x}), \tag{4}$$

which is linear by construction. In this equation, the choice of the potential $V(\vec{x})$ is usually determined by a number of considerations. In this section we will give some arguments for how nonlinearities can arise – despite the original linearity of the formalism.

Exactly and partially solvable models

Let us consider as an example the exactly solvable (ES) harmonic oscillator $V_{(HO)}(\vec{x}) \sim |\vec{x}|^2$, which has the equidistant, purely vibrational spectrum of the low-lying bound-state energies $E_{(HO)}$ and the localized wavefunctions of the form:

$$\psi_{(HO)}(\vec{x},t) \sim \exp(-iE_{(HO)}t)\exp\left(-|\vec{x}|^2/2 + O(\ln|\vec{x}|)\right), \tag{5}$$

which can be regarded as a natural realization of the above-mentioned Madelung decomposition. The other choices of a potential can be considered on grounds of either the formal relevance of $V(\vec{x}) \neq V_{(HO)}(\vec{x})$, such as the closed-form tractability of the ES Schrödinger equations [29], or its capability of describing the variability of the dynamics (by adding perturbations to $V_{(HO)}(\vec{x})$).

A robust connection between formalism and phenomenology can be also achieved via the quasi-exactly solvable (QES) models [30]. In this picture, one starts from a qualified guess of a few suitable trial wavefunctions, while the logarithmic corrections in (5) are replaced by an explicit ansatz. For example, in a case of an ℓ th partial wave and an n th radial excitation, the formula

$$\psi_{n,\ell}^{(QES)}(r,t) \sim \exp(-iE_n t)\exp(-r^2/2)\sum_{k=0}^{N} a_k^{(n)} r^{k+\ell+1}, \tag{6}$$

where $r = |\vec{x}|$, can be given as an illustration [31]; one can also consult Ref. [32] for the extension of this approach to constructing of quantum models.

In studies of the ES and QES models, an attention is paid to formal questions [33]. Here we intend to pursue a slightly different implementation of the ansatz idea: an accent will be moved from the details of usage of correction terms $O(\ln|\vec{x}|)$ in Eq. (5) to the universal large-$|\vec{x}|$ behavior of the time-independent magnitude of the asymptotically harmonic wave functions, as described by formula (5),

$$\psi_{(HO)}^{*}(\vec{x},t)\psi_{(HO)}(\vec{x},t) \sim \exp(-|\vec{x}|^2 + O(\ln|\vec{x}|)). \tag{7}$$

Such an asymptotic behavior of wavefunctions may be perceived as related to a certain equivalence-class property of potentials,

$$V_{(initial)}(\vec{x}) = |\vec{x}|^2 + O(\ln|\vec{x}|) \sim -\ln[\psi^*(\vec{x},t)\psi(\vec{x},t)] = V_{(generalized)}(\vec{x}), \tag{8}$$

and similar relation could be derived from the QES model (6):

$$V_{(generalized)}(r) = -b\ln[\psi^*_{(QES)}(r,t)\psi_{(QES)}(r,t)] \sim r^2 + O(\ln r) = V_{(initial)}(r), \tag{9}$$

if one imposes $b = 2$. It is thus not too surprising that the logarithmically nonlinear Schrödinger equation (1) was proposed as a "minimal" generalization of its linear predecessor in quantum theory [1].

Notice that the correction term $O(\ln|\vec{x}|)$ is negligible in the asymptotic domain only, while the non-exponential components of the wavefunction dominate in the vicinity of the origin, cf. Eq. (6). For this range of coordinates, Eq. (7) becomes replaced by an alternative estimate, $\psi^*(\vec{x},t)\psi(\vec{x},t) \sim O(|\vec{x}|^{const})$, which provides a limitation of the analogy between logarithmic and linear models.

Coupled-cluster method

Phenomenological success of the conventional (linear) Schrödinger equation is supported by employing various sophisticated methods of solving, such as the so-called $\exp S$ approach, known also as the coupled-cluster method (CCM) [34]. In this approach, the search for the wavefunction $\psi(\vec{x},t)$ can be performed by means of the operator premultiplication ansatz [35, 36]:

$$\psi = A\psi_0, \tag{10}$$

where ψ_0 is some trivial time-independent Slater-determinant reference function multiplied by an *ad hoc* operator A; the latter can be written in the exponential form $A = \exp\hat{S}$. In the field of numerical computations, this ansatz is often called a "preconditioning" of wavefunction [37].

The CCM construction of a specific physical state ψ is basically the reconstruction of the logarithm $\hat{S}$, via Eq. (4), in the form of its infinite-series expansion in a suitable basis of creation and annihilation operators. There exist several reasons for the practical success of the CCM approach in which one replaces the construction of the Hilbert-space vector ψ by that of the operator, i.e., the $M \times M$ matrix A with $M \to \infty$. Among them, let us mention here just the fast *a posteriori* convergence of computations, via finite-dimensional truncations $M < \infty$, which happens in the applications related to quantum chemistry.

For most of the practical applications of CCM, one assumes a stationary system and constant ψ_0, then Eq. (4) reduces to a time-independent eigenvalue problem. The latter can be rewritten in the linear form

$$\left(\hat{H} - E\right)\exp\hat{S} = 0 \tag{11}$$

and can be satisfied by suitable preconditioning operators $\hat{S} = \ln A$, see Ref. [38] for more details.

3. Theory of quantum liquids

In this section, we show how LSE appears in a theory of dense quantum liquids, with further applications to helium-4 superfluidity, theory of Bose-Einstein condensation (BEC), theory of physical vacuum, gravity and high-energy physics.

Classical liquids are discrete fluids consisting of localized atoms or molecules which thermal de Broglie length is smaller than an average separation between atoms or molecules. At the room temperature, this length is rather small: for example, for a water molecule it is less than the radius of the hydrogen atom. Therefore, classical liquids can be considered as continuous only on a scale which is much larger than the above-mentioned separation. In quantum liquids it is all different: atoms become delocalized because the de Broglie lengths are always larger than the inter-atomic distance, hence they overlap with each other. Thus, quantum liquids must be described in terms of fluid volume elements (also called the fluid parcels in the Lagrangian description), rather than in terms of discrete atoms. Then non-locality and continuity are preserved for very short scales of length.

However, few questions remain open in this regard. Firstly, it is the interplay between original degrees of freedom (for instance, He-4 atoms) and emergent collective ones - such as the fluid volume elements/parcels. Since the fluid-dynamical description of liquid helium presumes the usage of the volume element's notion, the latter must be physically justified: while for classical fluids the continuity is just the long wavelength approximation, for quantum liquids it is an empirically established fact. Therefore, searches for the most suitable collective degrees of freedom in a theory of superfluids continue.

Bose-Einstein condensation is yet another aspect of the superfluidity phenomenon which matters. The existence of BEC in the superfluid phase has been confirmed by inelastic neutron scattering, thus, the influence of BEC upon the properties of liquid helium is a subject of current studies as well.

If one accepts that strongly correlated helium atoms can form a bound state characterized by a single macroscopical wavefunction then the wave equation describing such object cannot be of the Gross-Pitaevskii (GP) type, also known as the cubic Schrödinger (in the relativistic field theory, its analogue would be the quartic scalar field theory ϕ^4). There exist at least two reasons for this. Firstly, the GP approach is a perturbative approach, which takes into account only two-body interactions and neglects anomalous contributions to self-energy, which is a robust approximation for dilute system like cold gases. However, it is unlikely to be sufficient for more dense objects like liquids: according to aforesaid quantum liquid's atoms are delocalized and thus nothing prevents them from getting involved into multiple-body interactions. One example of why multi-body (three and more) interactions are very important for forming bound states of bosons at low temperatures is the Efimov state, which has been experimentally observed in helium. The second issue is that the ground-state wavefunction of the GP BEC model in absence of an external potential trap does not describe a localized object. Instead, the free GP condensate tends to occupy all available volume - as such one needs to apply some kind of a trap, in order to confine the condensate and stabilize the system. This feature is more pertinent to gases than to liquids.

Luckily, there exists another candidate where the above-mentioned issues simply do not occur in the first place. This is nonlinear Bose liquid defined by virtue of the logarithmic Schrödinger equation of the type (1), except that the condensate wavefunction is normalized not to one but to a total amount of condensate particles, see Refs. [23, 19, 20, 21]. The logarithmic Bose liquid has a number of features suitable for our objectives: it implicates not only binary but also multiple-body interactions (when three or more bodies can scatter simultaneously), and its ground state is the so-called gausson – a spherically-symmetric object which is localized and stable even in absence of a trapping potential, with the interior density obeying the Gaussian law

$$\rho = |\psi(\vec{x},t)^2| \sim \exp(-|\vec{x}|^2/a^2), \tag{12}$$

where a is a characteristic size. Notice that this object is different from the classical droplet because it does not have border in a classical sense: its stability is supported by nonlinear quantum effects in the bulk, rather than by surface tension.

In the work [20], an analytical theory of structure and excitations in superfluid helium was proposed, which elaborates on above-mentioned microscopical aspects and goes beyond the GP approximation. It consists of two nested models which act on different length scales, but are connected via the parametric space: quantities and values of parameters in the long-wavelength model are derived from the short-wavelength part. The short-length model justifies appearance of collective degrees of freedom that can be used for describing the volume elements (fluid parcels) of the quantum liquid. Thus, an intrinsic structure of a fluid parcel is described using the non-perturbative approach based on the logarithmic wave equation: one can show that the interior density of the element obeys the Gaussian distribution. The long-wavelength model is the quantum many-body theory of the volume elements as effectively point-like objects, while their spatial extent and internal structure are taken into account by virtue of the nonlocal interaction potential. This potential is not postulated but derived from the short-wavelength part.

Furthermore, the quantitative agreement with different experimental data has been achieved: with only one essential parameter, there was reproduced at high accuracy (better than three per cent) not only the local ("roton") minimum but also the adjacent local ("maxon") maximum. The velocity of sound and structure factor were also computed and found to be in a very good agreement with experiment.

The fact that the logarithmic model was so instrumental in describing one of the realistic quantum Bose liquids we know of, gives us a hope that it can be also useful in a theory of physical vacuum. According to the latter, the physical vacuum is described by an essentially non-relativistic superfluid, while the Lorentz symmetry and relativistic gravity emerge in the linearized ("phononic") regime. This can be shown, independently, by means of the modular group approach [22], fluid/gravity correspondence [23], Bogoliubov method [19], and Arnowitt-Deser-Misner formalism [25]. Furthermore, there already exist some experimental data suggesting a connection between the phenomena of superfluidity and gravity [42, 43]. As a matter of fact, the superfluid vacuum paradigm allows us to explore a border of the relativity's applicability range, which is crucial for truly understanding of the spacetime approach as a physical theory, as well as to go beyond that border, both in a gnoseological and technological sense [24]. Moreover, it also reformulates general relativity and high-energy particle physics as branches of the quantum condensed matter theory and theory of open quantum systems, which makes knowledge more concise and systematic and greatly facilitates further studies, such as those towards constructing a theory of quantum gravity and unification of interactions.

4. Conclusions

In this essay, the logarithmically nonlinear quantum wave equation is introduced and discussed based on motivations coming from different areas of quantum physics. In the realm of the effective nonlinear quantum theories (i.e., say, for open quantum systems or in the theory of superfluidity or Bose–Einstein condensates [19]), the logarithmic model might be expected to provide numerous applications. Beyond that realm, the model is capable to play several other roles ranging from one of the specific nonlinear generalizations of the classical wave equation up to some of their non-classical, perturbative or non-perturbative extensions.

Besides, the model could offer new inspiration to the current experimental studies in quantum optics (cf. [8]) or to the methodical considerations in information theory (cf.

[13] - [18]). One of recent results by Babin and Figotin [44] reveal that in the semi-classical range, the combination of the logarithmic nonlinearities with the coupled-equation structures may prove unexpectedly productive. In the quantum limit of their formalism, the energy levels of the hydrogen atom tend to their quantum textbook values. At the same time, a contact between the classical and quantum phenomenology is achieved via physical reinterpretation of wavefunctions.

Other directions of work could include studies of mathematical structure of LSE with different external potentials or additional nonlinear terms, which can easily find applications in theory of quantum liquid mixtures, BEC excitations, vortexes, bosenovas, *etc*.

Acknowledgements

Research of K.Z. is supported by the National Research Foundation of South Africa under Grants No. 95965, 98083 and 98892. Research of M.Z. is supported by Grants GAČR 16-22945S and IRP RVO61389005, and his visit to Durban University of Technology was funded by the National Research Foundation of South Africa under Grant No. 98892.

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Received 15.12.2016