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Is pseudo-Hermitian quantum mechanics an indefinite-metric quantum theory? (English summary)

Quantum groups and integrable systems.
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Quantum mechanics working with systems of a finite number of degrees of freedom is usually considered incompatible with relativistic kinematics. In particular, the free Klein-Gordon equation is usually understood as one of the building blocks of a relativistic quantum field theory rather than as an eligible quantum-mechanical candidate for a sufficiently consistent description of a relativistic motion of a single spinless particle. One of the most immediate reasons lies in the non-Hermitian (in fact, in the present language, σ_3-pseudo-Hermitian) character of the corresponding one-particle time-evolution generator H (i.e., the energy operator, usually called the Feshbach-Villars Hamiltonian). Obviously, some states (one usually calls them antiparticles) may have negative norm so that, formally, the metric in Hilbert space does not exist (or, in other words, σ_3 remains indefinite and may merely be called a pseudo-metric). In the language of physics one simply cannot avoid a spontaneous creation of indefinitely many particle-antiparticle pairs in some sufficiently strong external fields.

Putting the main emphasis on the author's own contribution to the recent intensive development of the possible alternative physical interpretation of all the similar pseudo-Hermitian quantum-mechanical systems (also known as PT- or CPT-symmetric quantum mechanics; see, e.g., all of the papers in issues 1 and 10 of volume 54 of the Czechoslovak Journal of Mathematics for complementary reading), this review article tries to throw new light on many of the above-mentioned 75-year-old questions. The author describes an alternative interpretation of the free one-particle Klein-Gordon system with real spectrum (i.e., in the absence of any strong external field). His proposal (with details available in some of his recent publications) is based on the introduction of a new scalar product (the only formula in his text) reflecting the existence of a nontrivial positively-definite metric $\eta_+ \neq I$ (factorized as CP in a major portion of the literature) in Hilbert space. This makes the Feshbach-Villars Hamiltonian H (quasi-)Hermitian and endows Klein-Gordon states with a new eligible consistent probabilistic interpretation. *M. Znojil* (Řež u Prahy)