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**Review text:**

Even if the spectrum of energies (say, of bound states) is kept real (basically, by the requirements of common sense), one may freely try to extend the class of admissible Hamiltonians beyond their essentially self-adjoint “obligatory” form. Such a trick is known in relativistic quantum mechanics where, according to H. Feshbach and F. Villars [Rev. Mod. Phys. 30 (1958) 24] the Klein-Gordon equation may be given Hamiltonian formulation in the Hilbert (or rather Krein’s) space using an indefinite metric for scalar products. The same trick has been used in nuclear physics in the seventies, eighties and nineties when one of the preferred forms of the Hamiltonian in the so called interacting-boson model was non-Hermitian [so called quasi-Hermitian, see F. G. Scholz, H. B. Geyer and F. J. W. Hahne, Ann. Phys. (NY) 213 (1992) 74]. Finally, a re-incarnation of the trick characterizes a fresh wave of interest in the non-Hermitian Hamiltonians with real spectra. This wave was evoked by Bender and Boettcher in the theoretical quantum mechanics in the late nineties (cf. ref. [1]) and is represented also by the letter in question.

In the latter context, many recent papers addressing the community of physicists are re-examining the foundations of the standard quantum mechanics. The use of Hamiltonians which are not essentially self-adjoint encounters the most dramatic development. Probably the most influential mathematical step was made by Ali Mostafazadeh in 2002 [cf. ref. [17] which already appeared in J. Math. Phys. 43 (2002) 43] who reformulated and extended the Bender’s and Boettcher’s “PT symmetric quantum mechanics” to a theory which works with pseudo-Hermitian Hamiltonians.

In this letter by Z. Ahmed, the puzzling existence of the strictly real spectra

of energies generated by non-Hermitian Hamiltonians is discussed via explicit examples supporting the latter approach. The important role of the so called gauge transformation is discovered and emphasized. It is shown that and how the transition from PT symmetry to pseudo-Hermiticity clarifies the work with Hamiltonians containing components of a magnetic field.